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UNSTEADY PROCESSES IN AEROSPACE POWER UNITS WITH STAGE SEPARATION†

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The process of the ejection of the body of the booster from the duct of a ramjet which is used as a sustainer is simulated. The booster body is ejected by the gas dynamic free stream after the propulsion burnout. A mathematical description is proposed of the motion of the booster body and the accompanying gas dynamic flow in the freed part of the duct with the subsequent establishment of a flow before the starting of the combustion chamber of the sustainer. The problem reduces to the simultaneous integration of the equations of unsteady gas dynamics in a duct, the form of which changes with time, and the equations of motion of a rigid body under the action of forces which depend on time and are obtained during the solution of the gas dynamic problem. Fundamental factors are revealed which influence the ejection of the booster body, a domain of safe separation of the stages is constructed in the space of the governing parameters, the dynamic loads on the walls when the flow is being established are determined, and the steady state in the duct before the starting of the combustion chamber is calculated. The time taken for the stages to separate, which is needed in the solution of the trajectory problem, is calculated. The mathematical model also takes account of the change in the ramjet duct due to the construction of the critical cross-section of the jet, the introduction of flame stabilizers, and so on.

The study of the unsteady flows which accompany the ejection of a booster is important in the development of the construction, trials and modernization of ramjets. In addition to those being considered, unsteady flows in the duct of the ramjet during its motion along a trajectory also occur during the starting of the combustion chamber, when there is a change in the fuel supply conditions, when a shock wave is incident on the inlet to the ramjet and when there are other actions. Some of these have been considered in [1].

1. FORMULATION OF THE PROBLEM

The location of the booster body and its motion along the ramjet duct are shown in Fig. 1. In the majority of constructions, the butt of the booster body in its initial position completely overlaps the cross-section of the air intake, and the booster body rests on the mountings (n_i, m_j), located in the intermediate duct between the air intake (AI) and the combustion chamber (CC) (Fig. 1a).

The booster body begins to move at the instant when the force due to pressure on the butt becomes equal to the thrust and the friction force on the mountings. Until the buster body leaves its mountings, its motion is rectilinear, but, after passing the upper mountings, a rotation of the body around the last lower mounting is imposed on the rectilinear motion, as a result of which a collision can occur between the booster body and the internal walls of the ramjet duct (Fig. 1b) causing damage to these walls.

The problem therefore arises of determining the range of flight parameters (height, Mach number, vertical overloads, angle of attack, etc.), where separation of the stages occurs without collision which can lead to an accident. The optimal conditions for the ejection of the booster body must therefore ensure its linear motion within the ramjet duct. It should be added that, as the booster body moves out from the duct, forces start to act on it which are caused by the flow around the part of the body which has moved out. The magnitudes of these forces are frequently obtained experimentally.

The motion of the booster body along the ramjet duct accompanied by the filling of the freed volume and the gaps between the walls of the booster and the ramjet duct by high pressure gas and by the development of a flow in it.

We shall consider the flow which arises in the ramjet duct, beginning from the instant of time when the booster body moves from its initial position and, simultaneously, the motion of this body under the action of various forces. We direct the x axis along the axis of the ramjet duct, assuming that the duct

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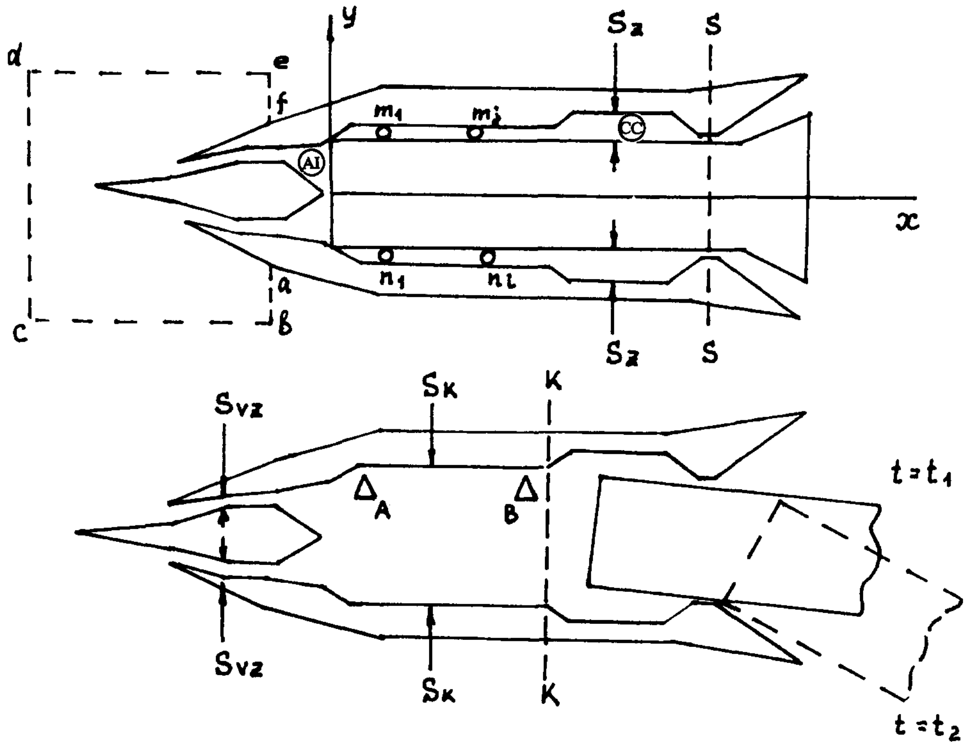

Fig. 1.

is axially symmetric, the y axis is perpendicular to the x axis in the meridian plane and the z axis is perpendicular to the meridian plane. The origin of coordinates is placed in the butt plane of the booster in its initial position.

We shall write out the equations of motion of a perfect, inviscid gas in the ramjet duct and, also, the equations of motion of the booster body, which after it has been released from constraints, reduce to the description of the translational motion of the centre of mass as a point mass and rotation about the centre of mass and rotation about the centre of mass [2]

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{U}) = 0, \quad \rho \frac{\partial \mathbf{U}}{\partial t} + \rho(\mathbf{U}\nabla)\mathbf{U} + \nabla p = 0 \tag{1.1}$$

$$\frac{\partial \rho e}{\partial t} + \operatorname{div}[\mathbf{U}(\rho e + p)] = 0; \quad e\frac{|\mathbf{U}|^2}{2} + \varepsilon(p,\rho), \quad \mathbf{U} = \{u, \upsilon, w\}$$

$$\frac{d\mathbf{r}}{dt} = \mathbf{V}, \quad \frac{d\varphi}{dt} = \omega, \quad m\frac{d\mathbf{V}}{dt} = \mathbf{f}, \quad J\frac{d\omega}{dt} = M_c, \quad \mathbf{r} = \{X, Y\} \tag{1.2}$$

$$\mathbf{f} = \{f_x, f_y\}, \quad \mathbf{V}\{V_x, V_y\}, \quad f_x = \{[F - R(t)]\cos\varphi + \tag{1.3}$$

$$+(E_x - Q)\cos\alpha - N_n(\sin\varphi + k_n\cos\varphi) - k_m N_m - E_y \sin\alpha +$$

$$+n_y P_g \sin(\alpha + \theta)\}, \quad f_y = \{[F - R(t)]\sin\varphi + (E_x - Q)\sin\alpha +$$

$$+N_n(\cos\varphi - k_n \sin\varphi) - N_m + E_y\cos\alpha - n_y P_g \sin(\alpha + \theta)\}$$

$$M_c = \{(k_n N_n \sin\varphi - N_n\cos\varphi)(X - X_n) - (k_n N_n \cos\varphi + N_n \sin\varphi)\times$$

$$\times(Y - Y_n) + N_m(X - X_T) + k_m N_m(Y_m - Y) + (E_x \sin\alpha +$$

$$+E_y\cos\alpha)(X_b - X) + (E_x\cos\alpha - E_y\sin\alpha)(Y - Y_b)\}$$

Here, ε, $\mathbf{U}$, p, ρ are the internal energy, the velocity vector, the pressure and density of the gas, $\mathbf{r}$ and $\mathbf{V}$ are the radius vector and the velocity vector of the centre of mass of the booster body, φ and ω are the angle between the axis of the booster body and the x axis and the angular velocity of rotation of the booster body about its centre of mass, m is the mass of the booster body, J is the moment of inertia of the booster body with respect to its centre of mass, $F = pS_T$ is the force due to the gas pressure on the buster butt, S_T is the butt area, $R(t)$ is the law of the change in the booster thrust due to the combustion of the fuel, $\mathbf{E} = (E_x, E_y)$ is the aerodynamic force, acting on the moving-out part of the booster and is proportional to the area of the part which has moved out and to the dynamic head of the free stream on it, Q is the inertial force due to the rocket deceleration in the atmosphere and the non-inertial character of the system of coordinates in which the booster motion is being treated, N_n and N_m are the reaction forces of the lower $(n_1, \dots, n_i)$ and upper $(m_1, \dots, m_j)$ controlling mountings (Fig. 1a), k_n and k_m are the corresponding friction coefficients, P_g is the gravitational force, n_y is the coefficient of vertical overloading, α is the angle of attack, θ is the angle of inclination of the velocity of motion of the aircraft to the horizontal, X_n, Y_n are the coordinates of the last of the lower mountings (n_i), X_m, Y_m are the coordinates of the last of the upper mountings (m_j) (Fig. 1a), X_T is the abscissa of the upper edge of the booster butt, and X_b, Y_b are the coordinates of the centre of mass of the part of the booster body which has moved out.

The quantities $\mathbf{E}$, Q, N_n, N_m contain empirical coefficients which are determined experimentally. The quantity J can be calculated from the shape of the booster body.

It is necessary to specify the initial and boundary conditions for the integration of system (1.1)–(1.3). The instant of time t_0 corresponds to the onset of the motion when the force f_x vanishes due to a reduction in the booster thrust. In this case

$$\mathbf{V}(t_0) = 0, \quad \varphi(t_0) = \omega(t_0) = 0, \quad \mathbf{r}(t_0) = \mathbf{r}_0 \tag{1.4}$$

The quantities V_y, φ, ω remain equal to zero until the centre of gravity of the booster body crosses the last lower mounting and the booster butt passes the last upper mounting. The last upper mounting is always located to the left of the final lower mounting in order to avoid jamming of the booster body (Fig. 1a).

The initial distributions of the gas dynamic quantities are determined from the solution of the problem of the flow around an aircraft with an air intake covered by the booster (Fig. 1a).

The computational domain for the system of equation of gas dynamics (1.1) includes the part of the ramjet duct not occupied by the booster from the inlet to the air intake up to the booster nozzle and the domain of the external flow *abcdef* before the inlet to the air intake (Fig. 1a). Inclusion of the external flow in the computational domain enables one to take account of the motion of the leading shock wave initiated by the gradual freeing of the air intake after the beginning of the booster motion.

Supersonic free-stream parameters are specified on the left boundary of the computational domain (the line *cd* in Fig. 1a). No-flow conditions, which take account of the velocity of the booster motion, are imposed on the solid walls, and the condition of constancy of pressure, which is preserved while the local gas velocity remains subsonic ($M_s < 1$) and is replaced by a supersonic outflow condition when $M_s > 1$, is imposed at the jet exit. Conditions for there to be no reflection of perturbations are imposed in segments *cd* and *de* of the domain *abcdef* (Fig. 1a) and supersonic outflow conditions are imposed on the lines *ef* and *ab*.

The calculation is continued in time until the booster butt passes across the critical cross-section of the jet and a flow is established in accordance with the flight conditions and the flow-through part of the duct which has been formed (the flame stabilizers are lowered during the release of the combustion chamber and, after the freeing of the critical cross-section of the jet, it can change its area).

The need for the simultaneous solution of Eqs (1.1)–(1.3) is a special feature of the problem, since the pressure p occurs in expression (1.3) for the forces and moments of the forces acting on the booster, and the boundaries of the computational domain and the boundary conditions for system (1.1) change in accordance with the booster motion. The latter fact makes the numerical algorithm far more complicated as the motion (especially the rotation) of the booster can strongly deform the computational mesh which, from considerations of the accuracy and convenience of the calculations, must be associated with the booster surface. We also emphasize that, in spite of the axial symmetry of the ramjet duct, the gas flow is spatial by virtue of the rotation of the booster body with respect to its centre of mass and the need to take account of the angle of attack.

We now present some considerations and estimates which simplify the problem.

First we note that, in the constructions being considered, the ratio of the maximum area of the gap S_z between the booster body and the walls of the ramjet duct to the area of the critical cross-section of the air intake S_{vz} does not exceed 20% and the ratio of S_z to the characteristic area of the cross-section of the ramjet duct S_k does not exceed 10% (see Fig. 1a). From the onset of the booster motion, the gap between the booster walls and the duct walls is filled by high-pressure air and, in front of the inlet to the combustion chamber, where the ramjet duct expands (section kk in Fig. 1), a flow is established with a passage through the speed of sound which is terminated by a closing shock wave. In the case of a sonic flow of the gas in the gap between the walls of the booster and the ramjet duct, the Mach number M_k in the freed part of the ramjet duct, which is estimated using the formula

$$\frac{S_z}{S_k} = M_k\left[\frac{\gamma+1}{2+(\gamma-1)M_k^2}\right]^{\xi}, \quad \xi = \frac{\gamma+1}{2(\gamma-1)}$$

is of the order of 0.1, and the Mach number of the throat of the air intake $M_v \sim 0.2$. Consequently, for the pressure change in front of the booster caused by the outflow of gas into the gap during the booster motion, we have

$$\frac{\Delta p}{p} \sim 1-\left(1+\frac{\gamma-1}{2}M_k^2\right)^{\xi} \sim 7\times 10^{-3}$$

An exception is the comparatively short time interval (less the 10% according to the calculations) when the booster butt enters into the combustion chamber (Fig. 1b). The onset of the release of the combustion chamber volume is accompanied by a brief drop in the pressure by 20–30% (see curve B in Fig. 2 which demonstrates the pressure change in section KK during the whole of the time that the booster is moving along the ramjet duct). The pressure drop caused by entering the booster butt into the combustion chamber occurs at the instant of time t_1. At the same time, the volume of the combustion chamber is filled with high-pressure air and the critical cross-section of the part of the ramjet duct which is free from the booster is displaced from the cross-section KK to the critical cross-section of the jet SS. A pressure drop in front of the booster butt also occurs due to the increase in the velocity of the booster motion with time. Estimates and calculations show that the maximum velocity of the booster is such that the mean value of the Mach number in the part of the ramjet duct which is freed does not exceed $M_k \sim 0.2$ and the pressure is reduced by $\sim 3\%$. A sharp reduction in the pressure in the ramjet duct is only observed after the booster passes the critical cross-section of the jet, completely freeing the duct (Fig. 1b). However, the study of the booster motion after it has crossed the critical cross-section of the jet is of no interest from a practical point of view since it has no effect on the development of the flow in the ramjet duct. So, the gas velocity and the pressure gradients in the section between the air-intake inlet and the booster butt remain small during the whole of the stage separation process.

A second important feature of the problem under consideration lies in the fact that the angle of rotation φ and the rate of rotation of the booster remain small during the motion. This is associated with the fact that, up to the instant of crossing the last upper mounting, after which it also beginning to rotate, the booster succeeds in gathering a significant longitudinal velocity, so that the time of its motion up to the critical cross-section of the jet is a small part of the overall time of motion along the duct.

As was pointed out above, the supercritical pressure drop between the retarded flow in front of the booster butt and the atmospheric air across the section of the jet creates a transonic flow in the gaps between the booster body and the walls of the intermediate duct of the ramjet. The booster traverses a large part of the intermediate duct while resting on its mountings, that is, without rotation. As calculations show, the angle of rotation of the booster is very small up to the instant when the booster butt enters the combustion chamber. Hence, the non-uniformity in the pressure field, which is created due to rotation and acts on the lateral surface of the booster, is quite unimportant. In the combustion chamber where the area of the gaps increases by almost an order of magnitude and the gas velocity falls, the effect of non-uniformity can also be neglected.

The estimates presented show that, in calculating the gas flow in the ramjet duct, the booster motion can be considered as one-dimensional and the flow itself as being axially symmetric (for zero angle of attack α).

Problem (1.1)–(1.3) is therefore "split" into three much simpler problems.

Problem 1. The gas flow and the rectilinear motion of the booster are simultaneously calculated up to the instant when the booster butt crosses the critical cross-section of the jet. The left-hand boundary of the computational domain coincides with the air-intake inlet since, according to the estimates which have been made above, the gas velocity in the combustion chamber remains small at all instants of time and the displacement of the leading shock wave is insignificant. Allowing for the fact that the non-uniformity in the gas dynamic parameters across the section can be neglected, as was shown above, and that the length of the computational domain l is much greater than its characteristic diameter d ($l/d \sim 10/15$), the flow may be calculated in a quasi-one-dimensional approximation.

A quasi-one-dimensional approximation enables one to simplify the algorithm for solving the problem in a domain with mobile boundaries considerably. The law for the pressure change at the booster butt, the distribution of the gas dynamic quantities throughout the whole of the ramjet duct during the booster motion and, in particular, at the moment of the release of the critical cross-section of the jet as well as the dynamic loads on the walls of the ramjet duct are determined as the results of the solution.

Problem 2. The two-dimensional motion of the booster is calculated using the law for the pressure change $P_T(t)$ at the booster butt (using Eqs (1.2), (1.3)) and the domains for the safe separation of the stages are determined.

Problem 3. The establishment of a flow in the ramjet duct, caused by the abrupt opening of the critical cross-section of the jet, is calculated for the distribution of the gas dynamic quantities found from problem 1 at the instant when the critical cross-section of the jet is freed. In this problem, the computational domain includes the domain *abcdef* in front of the engine inlet and the freed, flow-through part of the duct up to the section of the jet. The dynamic loads on the walls of the duct and the flow parameters before the launching of the combustion chamber are determined. The problem is solved in the approximation of the two-dimensional transient Euler equations. Unlike in problem 1, the use of two-dimensional equations is necessary since the influence of the complex shape of the air intake (Fig .1) can affect the qualitative character of the flow which is established at supersonic velocities of flow into the engine. In addition, a correct description of the motion of the leading shock wave and, in particular, its approach into the air intake becomes possible.

2. RESULTS OF THE CALCULATIONS

Problems 1–3 were solved numerically using Godunov's method, Rodionov's method [4, 5] (problem 1), the fourth-order Runge–Kutta method with automatic selection of the step size (problem 2), and Rodionov's method once again (problem 3). Movable deformable meshes were employed when solving problem 1.

We shall consider problem 1. The pressure change with time $p(t)$ in the ramjet duct during the separation of the stages is shown in Fig. 2. The position of the points A and B, where the "pressure recorders" are located, is shown by the small triangles in Fig. 1(b). The dashed curve corresponds to the experimental data and the solid curve to the results of the calculations. The greatest differences ($\sim 10\%$) between the calculated and the experimental data, which are observed during the last phase of the booster motion, are associated with the fact that, in the calculation, the additional take off of air from the ramjet duct for the cooling system has not been taken into account.

The sharp increase in the pressure at the onset of the booster motion is explained by the fact that after a gap has been opened between the booster body and the walls of the ramjet duct, the gap is filled with high-pressure air which, when $t \leqslant 0$, had been in the air intake cover. The gas flow in the gap is close to sonic flow and, hence, the magnitude of the pressure in the gap $p_z \cong 0.5p_T$, where p_T is the pressure at the booster butt.

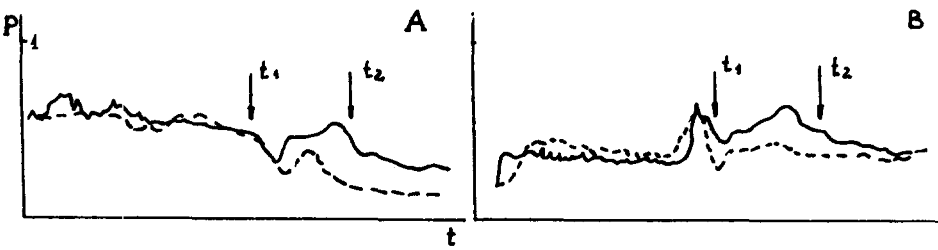

Fig. 2.

The pressure augmentation at point B at the instant before $t = t_1$ (on the right-hand graph of Fig. 2) corresponds to the passage of the booster butt across the section $x = B$ when the "pressure recorder" reaches a slow flow zone with an almost constant pressure p_T. The following brief pressure drop coincides in time with the start of the filling of the combustion chamber with high-pressure air after the booster butt enters the combustion chamber and the critical cross-section of the freed duct with allowance for the gaps is displaced into the nozzle throat. The pressure then again increases. This is associated with the termination of the filling of the combustion chamber with high-pressure air. The following monotonic pressure drop is a consequence of the increase in the booster velocity and, then, the freeing of the ramjet nozzle.

In certain constructions, vibrations, which evolve during the ejection process, develop in the air intake which is open towards the flow and blocked by the booster. The reasons for the occurrence of such vibrations in closed cavities has been described in detail (see [6–8] for example). The calculation of the pressure change with time at the butt of the booster during its motion along the duct taking account of the pressure oscillations developed in the closed air intake under the flight starting conditions shows that the time-averaged pressure barely changes and that the amplitude of the oscillations rapidly attenuates. This is associated with the fact that the booster motion changes the length and, together with it, the fundamental acoustic frequency of the cavity in which the oscillations occur or as a result of which the oscillations decay.

We now consider problem 3. After the opening of the jet, the wave process of the establishment of a flow in the duct, which is accompanied by the upstream propagation of a rarefaction wave, commences. The initial stage of this process, which ends when $t > t_2$, is also shown in Fig. 2. On crossing the throat of the air intake, the rarefaction wave reduces the pressure across the leading shock wave, which begins to move to the air-intake inlet. Simultaneously, there is an acceleration of the gas in the throat of the air intake leading to the formation of a local supersonic zone which culminates in a system of shock waves. The size of this zone depends on the Mach number of the free stream M and the geometry of the ramjet duct.

During its establishment, a closed shock wave can be stabilized either immediately in front of the combustion chamber or in a certain section of the air intake. If the leading shock wave does not succeed in entering into the air intake before a supersonic flow is established across the throat of the air intake, then it is stabilized in front of the air-intake inlet, and a flow is possible with two shock waves, one in front of the inlet and one in the air intake itself. In the opposite case, when the velocity of the shock wave overtakes the process of the establishment of a supersonic flow in the throat of the air intake, the leading shock wave travels into the ramjet duct and a supersonic flow is established behind it in the inlet section of the air intake. The occurrence of any particular form of the flow depends on the flight conditions (the height, Mach number, etc.) at the moment the stages separate.

The pressure distribution at different instants of time along the axis of the ramjet duct during the establishment of a flow with two shock waves is shown in Fig. 3.

Separation of the gas dynamic part of the problem enables us to carry out a multiparameter investigation of system (1.2), (1.3) and to construct a domain for the safe separation of the stages in the space of the governing parameters. An example of such a domain is shown in Fig. 4. The Mach number is plotted along the ordinate and the height of the flight H (in conventional units) along the abscissa. The upper boundary (AB) of the domain of safety is defined by the fact that, as the Mach number of the flight increases, the quantity $(p_T - p_H)$, where p_T is the pressure in the closed air intake and p_H is the atmospheric pressure, becomes larger. If $(p_T - p_H) > p^*$, which is determined by the strength characteristics of the construction, the engine may be damaged during the booster ejection by the pressure of the retarded flow. Hence, the line AB represents the curve which is defined by the relation

$$p_T(H, M_\infty) = p^* + p_H$$

When the Mach number M decreases (for a fixed H), the dynamic pressure at the booster butt drops, which leads to a reduction in the longitudinal velocity of the motion of the booster and increase in the ejection time. On the other hand, the angular velocity of the booster motion, which is mainly determined by the gravity force and the overloading coefficient n_y, only reacts weakly to a reduction in M_∞. As a result, the limiting angle of rotation φ of the booster body becomes larger as M_∞ decreases and attains a critical value at a certain M_∞, which corresponds to an impact of the booster on the wall of the ramjet duct. Hence, the relation M_∞^* $(H, n_y, \alpha, \theta, \ldots)$ defines the lower boundary of the domain of safe separation of the stages for specified values of the parameter $H, n_y, \alpha, \ldots$ In practice, separation can occur over a certain range of variation in the parameters $H, n_y, \alpha, \ldots$. The lower limit of the domain of safe separation of the stages in the (M_∞, H) plane is therefore determined from the relation

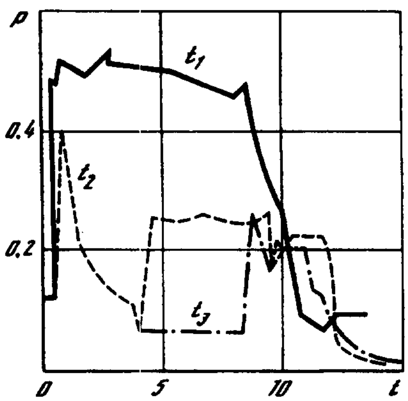

Fig. 3.

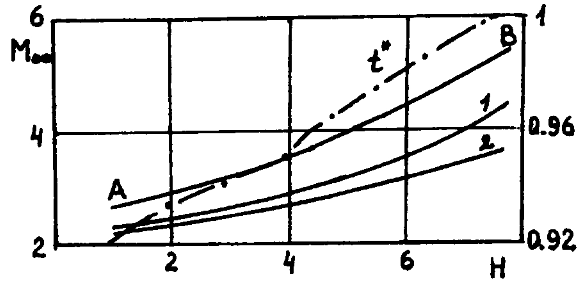

Fig. 4.

$$M^-(H) = \max\ M^*_\infty(H, n_y, \alpha, \theta, \ldots) \tag{2.1}$$

The arguments in (2.1) vary within the ranges

$$n_y^- \leqslant n_y \leqslant n_y^+, \quad \alpha^- \leqslant \alpha \leqslant \alpha^+, \quad \theta^- \leqslant \theta \leqslant \theta^+$$

As a rule, relation (2.1) is a piecewise-smooth curve consisting of segments of smooth curves $M^-(H, n_y, \alpha, \theta)$, where (n_y, α, θ) take any one of the boundary values.

The domain of safe separation of the stages also depends very much on the way the booster thrust $R(t)$ falls off, which is determined by the fuel composition of the launching stage and a number of parameters which characterize the preparation of the booster. The more rapid the thrust fall-off during the afterburning of the fuel, the more rapid increases in the longitudinal velocity of the booster and the lower the position of the curve (2.1).

It is also useful to know the instant when the flame in the booster is cut off during the afterburning of the fuel. Two laws for the thrust fall-off are presented in Fig. 5: $R_1(t)$ is the theoretical law and $R_2(t)$ is the experimental law. The instant when the flame is cut off ($t = 1$) is indicated in the case of R_2. The lower bounds of the domain for the safe separation of the stages (curves 1 and 2 for $R_1(t)$ and $R_2(t)$, respectively) have been plotted in Fig. 4 as given by these two laws. It is seen that the law $R_2(t)$ has a larger domain of safety than $R_1(t)$ and, furthermore, the instant when the flame is cut off, that is, the instant when the thrust vanishes, is known.

The time interval from the start of the motion of the booster body to the moment of the starting of the combustion chamber of the sustainer is an important parameter when solving trajectory problems.

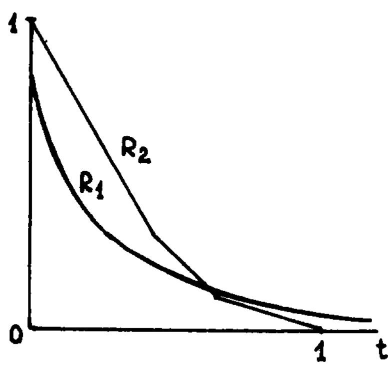

Fig. 5.

Over this time interval, the aircraft moves inertially (the thrust is equal to zero), its drag changes (the throat is opened and the ramjet duct is freed), the fuel supply system is switched on and the ramjet flame stabilizers are established, etc. The time interval is made up of the time of the booster motion along the duct, the time taken to establish a steady flow in the ramjet duct and the time required for the combustion chamber to reach the specified operational conditions. All these times are determined when solving the problems which have been presented in this paper, and the last one from [1].

The dependence of the time t^* (the scale is on the right) taken for the stages to separate on the height of the flight H, obtained along the lower boundary of the domain of safety in the case of the thrust law $R_2(t)$, is shown in Fig. 4. It is seen that the time required for the booster ejection increases with height, which is a consequence of the reduction in the dynamic head acting on the booster butt. Certain times t^* for the stages to separate can turn out to be unacceptable from the point of view of the trajectory problem (t^* is too large), and these constraints must also be taken into account when constructing the safety region.

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